Magnetoplasma waves in compensated metals with quasilocal electron states

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(Submitted December 21, 1992)
Fiz. Nizk. Temp. **19**, 406–409 (April 1993)

The effect of quasilocal electron states on transverse magnetoplasma waves in compensated metals is considered. It is shown that the coupling of magnetoplasma resonance with the electron resonance at an impurity results in the emergence of two branches in the magnetoplasma wave spectrum, which are separated by a forbidden frequency region (quasigap). The spectrum and damping decrement of the waves are calculated. The effect of the waves on the surface impedance of metals is considered. The resonance of impedance is observed at the upper boundary of the quasigap.

1. Magnetoplasma waves with a linear spectrum are observed in compensated metals in a strong magnetic field, when the Alfvén velocity exceeds the Fermi velocity of charge carriers.[1,2] The effect of magnetic-impurity electron states on the characteristics of these waves in the isotropic two-band model is considered in Ref. 3. It is shown that resonance electron transitions between magnetic-impurity levels and Landau levels (MIL → LL) lead to the emergence of a new limiting frequency in the wave spectrum, which is associated with the binding energy of an electron in the field of an impurity. Above this frequency, magnetoplasma waves cannot propagate in view of their strong resonance damping. On the other hand, resonance electron transitions from Landau levels to magnetoplasma levels or quasilocal levels (LL → MIL or LL → QL) lead to the emergence in the magnetoplasma wave spectrum of the forbidden frequency range similar to the quasigap in the phonon spectrum of the lattice with quasilocal vibrations.[4] Such transitions were not considered in Ref. 3. This region lies above the frequency of resonance transitions LL → QL. The group velocity of the waves vanishes at the upper boundary of the quasigap, leading to a resonance of the surface impedance of the metal.

In the present paper, we consider the results of calculations of the spectrum, damping decrement of transverse magnetoplasma waves, and the surface impedance of a metal with quasilocal electron states, viz., eigenstates[5] and magnetic-impurity states[6,7] in the vicinity of the frequencies of resonance transitions LL → QL and LL → MIL. We consider two groups of charge carriers (electrons and holes) whose constant-energy surfaces are in the form of ellipsoids of revolution with transverse $m_\perp^{(e,h)}$ and longitudinal $m_\parallel^{(e,h)}$ effective masses. The results are expressed in terms of quasilocal state characteristics, viz., the pole of the amplitude of the resonance electron scattering by an isolated impurity atom $\varepsilon_r - i\Gamma$ (ε_r is the position of the resonance and Γ its half-width), and the residue of the scattering amplitude at the pole r. The values of these quantities can be calculated by specifying the scattering potential, or determined experimentally. The magnetic field $\mathbf{H} \parallel \mathbf{z}$ is assumed to be oriented at angles θ_e and θ_h relative to the rotational axes of the electron and hole ellipsoids respectively.

2. The introduction of resonance electron scattering by isolated impurity atoms leads to the emergence of resonance terms associated with LL → QL electron transitions in the dynamic conductivity tensor $\sigma_{ik}(\mathbf{q}, \omega)$ ($\mathbf{q}$ is the wave vector and ω the frequency). In particular, in the vicinity of the frequency ω_s of LL → QL transitions, the resonance component σ_{xx} for the ellipsoid of revolution in the long-wave limit is found to be

$$\delta\sigma_{xx}^{(s)} = \frac{\omega_p^2}{4\pi\omega_s}\alpha_s i\left(\frac{\omega_s}{\omega - \omega_s + i\Gamma}\right)^{1/2}, \tag{1}$$

where

$$\alpha_s = \frac{m_\perp m_\parallel^{1/2}}{2^{3/2}\pi}\frac{n_i}{n_e}\frac{\Omega^2 r}{\omega_s^{5/2}} \times \left[f(\varepsilon_r - \omega_s) - f(\varepsilon_r)\right]\left[\frac{N-s}{(1-\Omega/\omega_s)^2} + \frac{N-s-1}{(1+\Omega/\omega_s)^2}\right] \tag{2}$$

is the energy of the resonance transition oscillator. Here ω_p and Ω are the plasma and cyclotron frequencies of electrons, n_e and n_i the number densities of electrons and impurity atoms, respectively, N is the number of Landau levels below ε_r, $\omega_s = \omega_0 + s\Omega$ are resonance frequencies, ω_0 the separation between ε_r and the lower-lying Landau level, $s = s_1, \ldots, s_2$ (s_1 is the number of Landau levels between ε_r and the Fermi boundary, and $s_2 = N - 1$), f is the Fermi function, and $\hbar = 1$. The root singularity in (1) is associated with the singularity of the electron density of states at the Landau level participating in resonance transitions, and the difference between Fermi functions in (2) takes the Pauli exclusion principle into account. Expression (1) is also applicable for transitions between Landau levels and magnetic-impurity levels (LL → MIL). The only difference is that now $\omega_s = s\Omega - \Delta$, where Δ is the electron binding energy,

$$r = \frac{2^{5/2}\pi\Delta^{3/2}}{m_\perp m_\parallel^{1/2}\Omega}, \tag{3}$$

and the oscillator energy (2) contains summation over the numbers of magnetic-impurity levels participating in transitions at a frequency ω_s. The resonance terms in the components σ_{yy}, σ_{xz} and σ_{zz} differ from σ_{xx} in the additional factors $m_\parallel/M$, $A = (m_\perp - m_\parallel)\sin 2\theta/2M$ and A^2 respectively. Here $M = m_\perp \sin^2\theta + m_\parallel \cos^2\theta$. The resonance contribution to $\sigma_{ik}(\mathbf{q},\omega)$ for MIL → LL transitions in the case of a quadratic energy–momentum relation for electrons was obtained in Ref. 3.

3. Let us consider the effect of quasilocal electron states on the spectrum and attenuation of magnetoplasma waves. It

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follows from the energy–momentum relation that for $\mathbf{q} \parallel \mathbf{H}$ there exist two linearly polarized waves whose polarization vectors are directed along the principal axes of the renormalized two-dimensional conductivity tensor.[2] Taking the resonance contribution into account, we can write the spectrum of the wave polarized along the y-axis in the long-wave limit in the form

$$q^2 = \frac{\omega^2}{v_a^2} + i\frac{\nu\omega}{v_a^2} - \frac{\omega_p^2}{c^2}\alpha_s\left(\frac{\omega_s}{\omega - \omega_s + i\Gamma}\right)^{1/2}, \tag{4}$$

where $v_a = H[4\pi n(m_1 + m_2)]^{-1/2}$ is the Alfvén velocity, $m_1 = (m_\perp m_\parallel/M)_e$; $m_2 = (m_\perp m_\parallel/M)_h$; $n = n_e = n_h$ the charge carrier concentration, ν the average relaxation frequency associated with potential scattering of electrons and holes by impurity atoms,[2] and c the velocity of light in vacuum. The first and second terms on the right-hand side of Eq. (4) contain the joint contribution of electrons and holes.[2] The singular term is associated with resonance electron transitions LL → QL. The range of applicability of Eq. (4) is determined by the inequalities

$$\nu,\, qv_F^{(e,h)} \ll \omega \ll \Omega,\quad v_F^{(e,h)} \ll v_a, \tag{5}$$

where v_F is the Fermi velocity of the charge carriers. These inequalities are valid for magnetoplasma waves for $s = 0$ with quasilocal electron eigenstates and $s = 1$ with magnetic-impurity states. Quasilocal electron states practically do not affect a fast magnetoplasma wave[2] polarized along the x-axis in view of anisotropy of the energy–momentum relation for electrons.

An analysis of Eq. (4) indicates that the coupling of the magnetoplasma resonance with the electron resonance at an impurity results in the emergence of two branches in the spectrum of magnetoplasma waves separated by the forbidden frequency range (quasigap) of width $\Delta\omega_s = \omega_s\alpha_s^2(\omega_p v_a/\omega_s c)^4$. The gap lies above the frequency of LL → QL resonance transitions. In the region $\omega > \omega_s$, the solution of Eq. (4) has the form $\omega_s(q) - i\gamma_s(q)$, where the energy–momentum relation for the waves and their decrement have the form

$$\omega_s(q) = \omega_s(0)\left[1 + 2\frac{\Delta\omega_s}{\omega_s}\left(\frac{qv_a}{\omega_s}\right)^2\right], \tag{6}$$

$$\gamma_s(q) = \Gamma + 2\nu\frac{\Delta\omega_s}{\omega_s}\left[1 + 3\left(\frac{qv_a}{\omega_s}\right)^2\right]. \tag{7}$$

The spectrum contains the limiting frequency

$$\omega_s(0) = \omega_s\left[1 + \alpha_s^2\left(\frac{\omega_p v_a}{\omega_s c}\right)^4\right],$$

lying at the upper boundary of the quasigap, where the group velocity of the wave is zero. The damping (7) of the wave is determined not only by the potential scattering of electrons and holes, but also by the width of the quasilocal level in the electron spectrum. The presence of small quantities Γ and ν in Eq. (7) ensures the smallness of the decrement as compared with the frequency (6). In the frequency range $\omega \lesssim \omega_s$, the wave spectrum is given by

$$\omega = \omega_s\left[1 - \frac{\alpha_s^2(\omega_p/c)^4}{4q^4}\right],$$

where

$$\omega_p\frac{v_F}{c}\alpha_s^{1/2} \ll qv_F \ll \omega_s.$$

In this region, a strong damping comparable with the resonance transition frequency ω_s is observed. Far away from ω_s, we can omit the singular term in (4), i.e., neglect the effect of quasilocal states on the characteristics of a magnetoplasma wave. The energy–momentum relation for magnetoplasma waves near ω_s is shown schematically in Fig. 1.

4. If the magnetic field $\mathbf{H}$ is perpendicular to the metal surface, the linearly polarized wave incident on the metal half-space attenuates according to the law[2]

$$T_{yy}(z,\omega) = i\frac{\pi}{2q(\omega)}\exp\,[iq(\omega)z], \tag{8}$$

where $q(\omega)$ is the solution of Eq. (4). The relevant contribution to the surface impedance for such a wave has the form

$$Z_{yy}(\omega) = 4\pi\omega/c^2 q(\omega).$$

The wave amplitude (8) as well as $Z_{yy}(\omega)$ have a resonance singularity at the limiting frequency $\omega_s(0)$ at which the group velocity of the wave is zero. Near the resonance, we have

$$Z_{yy}^{(s)}(\omega) = \frac{2^{5/2}\pi}{c^2}v_a(\Delta\omega_s)^{1/2} \times \left\{\omega - \left[\omega_s(0) - i\gamma_s(0)\right]\right\}^{-1/2}. \tag{9}$$

Root singularities of the impedance at frequencies $\omega_s(0)$ ensure the resonance excitation of magnetoplasma waves in metals with quasilocal electron states by an external electromagnetic wave. Figure 2 shows the dependence of the real and imaginary components of the quantity

$$\psi_1 = \frac{c^2\gamma_1^{1/2}(0)}{2^{5/2}\pi v_a(\Delta\omega_1)^{1/2}}Z_{yy}^{(1)}$$

on $\eta = \omega/\omega_1(0) - 1$ near the frequency ω_1 of resonance transitions LL → MIL. The value of ψ_1 is calculated by formula (9) for various orientations of the magnetic field relative

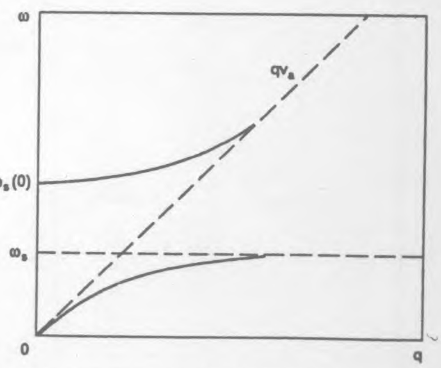

FIG. 1. Two spectral branches of magnetoplasma waves in metals with quasilocal electron states.

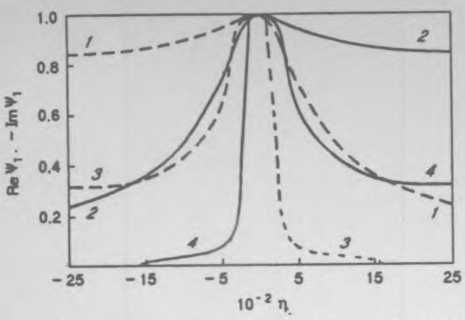

FIG. 2. Frequency dependence of the real (solid curves) and imaginary (dashed curves) components of the surface impedance for two magnetic field orientations near the resonance.

to the energy ellipsoids of electrons and holes in bismuth. Curves 1 and 2 (Fig. 2) were obtained for $\gamma_1(0)/\omega_1(0) = 1.25\cdot10^{-1}$, when the magnetic field is parallel to the bisector axis ($\theta_e = 0$, $\theta_h = \pi/2$), while curves 3 and 4 were obtained for $\gamma_1(0)/\omega_1(0) = 1.03\cdot10^{-2}$, when the magnetic field is parallel to the trigonal axis ($\theta_e = \pi/2$, $\theta_h = 0$). The calculations were carried out for the parameters of Bi spectrum[8] with donor impurities (e.g., Te or Se).[9] We assumed that $a = 10^{-6}$ cm is the scattering length, $n_i/n_e = 10^{-2}$, and $H = 10^4$ Oe. In this case, the dimensionless impedance $\zeta = c^2 Z_{yy}^{(1)}/4\pi v_a = 4$ for $\theta_e = 0$, $\theta_h = \pi/2$ and $\zeta = 13.9$ for $\theta_e = \pi/2$, $\theta_h = 0$.

5. In compensated metals in moderate magnetic fields, when $v_a << v_F^{(e,h)}$ and the role of spatial dispersion is significant, helical magnetoplasma waves can propagate under the condition that the wave vector **q** and the magnetic field **H** are parallel to the high-order symmetry axis.[10] In view of anisotropy of the energy–momentum relation for the charge carriers, their velocities and periods of revolution are different. For this reason, the high-frequency Hall conductivity of a compensated metal, taking spatial dispersion into account, does not vanish. For **q** ∥ **H**, Landau damping does not affect the transverse conductivity. In the long–wave limit, the dispersion equation for circularly polarized waves, on account of the resonance LL → QL transitions has the form

$$q^2 = \pm q^2 \frac{\omega}{\omega_r} + \frac{\omega^2}{v_a^2} + i\frac{\nu\omega}{v_a^2} - \frac{\omega_p^2}{c^2}\alpha_s\left(\frac{\omega_s}{\omega - \omega_s + i\Gamma}\right)^{1/2}, \tag{10}$$

where $\omega_r = cH/4\pi neR^2$ is the limiting frequency in the spectrum of the wave with the "plus" polarization,[10] and $R^2 = (1/5)|R_1^2 - R_2^2|$, ($R_1$, R_2 are the Larmor radii of the orbits for electrons and holes, respectively). The first term on the right-hand side of Eq. (10) emerges due to nonlocal corrections to the electron and hole components of the Hall conductivity. The range of applicability of Eq. (10) is defined by the inequalities

$$\nu << \omega << qv_F^{(e,h)} << \Omega\,; \quad v_a << v_F^{(e,h)}.$$

The introduction of resonance transitions LL → QL modifies the spectrum of the helical waves. For example, the spectrum of a wave with "minus" polarization consists of two branches of long-wave oscillations separated by a quasigap of the same width as for the linearly polarized wave considered above. The spectrum and attenuation of this wave in the region $\omega > \omega_s$ differ from (6) and (7) in the factor $(1 + \omega_s/\omega_r)$ in front of q^2. At the upper boundary of the quasigap, where the group velocity of the wave vanishes, the impedance component Z_- has a resonance singularity.[2] Near the resonance, the quantity Z_- differs from (9) in the factor $(1 + \omega_s/\omega_r)^{-1/2}$. Thus, the introduction of LL → QL transitions leads to the emergence of a resonance contribution to the impedance for a wave with the "minus" polarization, whose resonance excitation in conductors without quasilocal states is impossible.

The authors express their gratitude to N. M. Makarov and V. A. Yampol'skii for fruitful discussions of the results.

[1]S. Buchsbaum and J. Galt, *Phys. Fluids* **4**, 1514 (1961).

[2]É. A. Kaner and V.G. Skobov, *Usp. Fiz. Nauk* **89**, 367 (1966) [*Sov. Phys. Usp.* **9**, 480 (1967)].

[3]É. A. Kaner and A. M. Ermolaev, *Zh. Éksp. Teor. Fiz.* **92**, 2245 (1987) [*Sov. Phys. JETP* **65**, 1266 (1987)].

[4]A. M. Kosevich, *Theory of Crystal Lattice* [in Russian], Vishcha Shkola, Kharkov (1988).

[5]I. M. Lifshits, S. A. Gredeskul, and L. A. Pastur, *Introduction to the Theory of Disordered Systems* [in Russian], Nauka, Moscow (1982).

[6]A. M. Ermolaev and M. I. Kaganov, *Pis'ma Zh. Éksp. Teor. Fiz.* **6**, 984 (1967) [*JETP Lett.* **6**, 395 (1967)].

[7]A. M. Ermolaev, *Zh. Éksp. Teor. Fiz.* **54**, 1259 (1968) [*Sov Phys. JETP* **27**, 673 (1968)].

[8]V. S. Édel'man, *Usp. Fiz. Nauk* **123**, 257 (1977) [*Sov. Phys. Usp.* **20**, 819 (1977)].

[9]N. B. Brandt and L. G. Lyubutina, *Zh. Éksp. Teor. Fiz.* **52**, 686 (1967) [*Sov. Phys. JETP* **25**, 450 (1967)].

[10]E. A. Kaner and V. G. Skobov, *Phys. Lett.* **A25**, 105 (1967).

Translated by R. S. Wadhwa