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STUDY OF COUPLED NON-LINEAR SYSTEMS OF DIFFERENTIAL EQUATIONS LINKED BY NETWORK-ALIKE NEIGHBORHOOD

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Complex behavior of coupled ordinary differential equations is considered. The bonds between equations have network structure, including small-world type. Results of numerical experiments are described. Some cases of self-organization patterns are proposed.

Фачетті Л., Макаренко О. Розглянуто складну поведінку зв'язаних звичайних диференціальних рівнянь. Зв'язки між рівнянь мають структуру мережі, в тому числі малого світового типу. Наведено результати обчислювальних експериментів. Запропоновано деякі випадки самоорганізації в моделях.

Ключові слова: ЗВ'ЯЗАНІ РІВНЯННЯ, МЕРЕЖІ, 'МАЛИЙ СВІТ', САМООРГАНІЗАЦІЯ, МОДЕЛЮВАННЯ.
Keywords: COUPLED EQUATIONS, NETWPRKS, 'SMALL WORLD', SELFORGANIZATION, MODELLING.

I. Problem

This report studies the behavior of non-linear differential equations linked by network-alike connections. Earlier, these equations have been used to model the defects in a nuclear plant reactor. Before the equations usually had been considered on the lattice. Here we introduce other types of neighborhood, less regular than lattice. We had investigated the network structure impact on the outcomes of the differential equations.

II. General Model

In this section, we present the model of differential equations that we will solve numerically. Our objective is to solve numerically the following equations:

Differential system

$$\frac{\partial \varphi_i}{\partial t} = f_i(\varphi_1, ..., \varphi_n), i \in \{1, ...n\}$$

$$f_i(\varphi_1, ..., \varphi_n) = f(\varphi_i) + \sum_{j \neq i} \omega_{ij} f(\varphi_j)$$

$$f(u) = exp(\frac{u}{1 + \alpha u})$$

where: φ_i values are called the "elements chain", or "the chain";
ω_{ij} represents the value at row i column j of the Adjacency Matrix (it will be defined later) ; α is a real value that we will fix.
We make an analogy between our model of coupled differential equations and graph theory. First we consider that every function φ_i is represented by a node. We introduce an adjacency matrix M which models these edges between each cells. These edges are undirected.
The case where the graph is a ring, with each node linked to its two closer neighbors has already been simulated in scientific literature.
cells and 1 by black cells).
In our simulations, we intend to explore other types of networks, i.e. other types of adjacency matrix. It cannot be called a "small world" as the number of nodes composing our network is far too small (around 20 nodes) to be considered as a "large" network.

Initial conditions Initially, each element φ_i of the chain has value : $\varphi_i(t=0)$ =sin(a*i) **if sin(a*i) > 0 ;** 0.01 **if sin(a*i) <= 0.**
This function attempts to reach the physical reality of the defects.

III. Numerical experiments

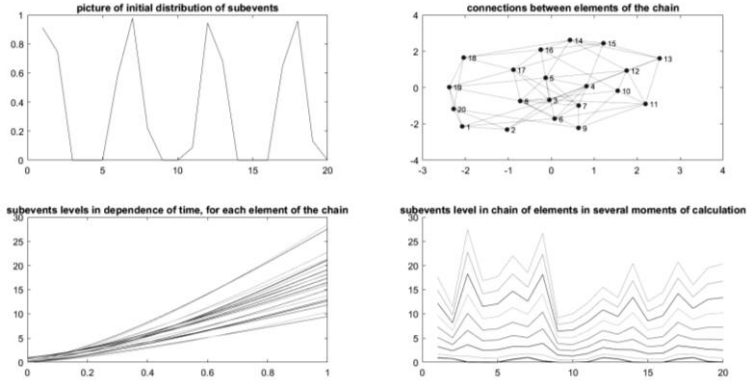
All the simulations were carried out with MATLAB Software.

Parameters set Small-world parameters : For the experiments, we used k=4, β=0.2. Initial conditions parameters : For the experiments, we used a=20. Hence, we have:

$\varphi_i(t=0) = \sin(20i)$ **if sin(20i) > 0 ;** 0.01 **if sin(20i) <= 0**
*α parameter of function f :*For the simulation, we used α=1.

Results

Here, we present four different simulations. On each simulation's results there is: in the top-left corner, the initial chain state; in the top-right corner, the network structure; In the bottom-left corner, the evolution of every function φ_i of the chain (represented by a different color) over the time. In the bottom-right corner, the state of the chain at several moments of time.



IV. Conclusions

The simulations led us to general conclusions regarding the usual behavior of the chain. All the chain elements have the same type of behavior, with increasing regular functions tending to affine function when time tends to infinite. These comportments are dependant of the initial conditions, which we fixed positive for every element of the chain. Then, when integrating these positive function over the time, we obtain increasing functions φ_i. Next, a basic study of f function – which tends to a constant value when its argument u tends to infinity – indicate us that the integrated function φ_i will converge towards