

Reliability Tensor Model of Telecommunication Network with RED

Olexandr V. Lemeshko, Oksana Yevsyeyeva, Sergey Garkusha

Abstract—If the user is buying a telecommunications service, he expects from the provider of a certain quality of servicing. So the delivery of telecommunications traffic must be realized with the specified quality requirements. As a rule the requirements are related to traffic rate, average delay, packet jitter, and the reliability (delivery or loss probability). The main way to meet it is finding appropriate path or multipath along which these requirements are satisfied. The multipath case is related to traffic distribution task. In this article we proposed tensor model for telecommunication network with RED and formulated analytical condition for QoS-ensuring. Satisfaction reliability of the condition guarantees that rate and requirements are achieved at the same time. The formulated condition has invariant form that doesn't depend on AQM mechanism type.

Index Terms—Delivery Probability, QoS, Packet loss, RED, Reliability, Telecommunication Network, Tensor model

I. INTRODUCTION

GUARANTEED end-to-end Quality of Service (QoS) is one of fundamental aspects of modern telecommunication network (TCN) [1]. In practice QoS ensuring is related to different traffic control features such as classification and marking, routing, shaping, policing, queuing, congestion management [2]. From viewpoint of network productivity the most effective QoS-features from the list are multipath routing as tool for load balancing and Active Queue Management (AQM) mechanisms as tool for congestion management.

In general the end-to-end QoS requires guaranteeing on multiple QoS-parameters, such as rate, average delay, packet jitter, and the reliability (delivery or loss probability) at the same time. As result QoS ensuring is complex and difficult task that needs appropriate mathematical models and algorithms. Currently within the existing technological traffic control means (protocols and mechanisms) the routing and

AQM problems are solved apart by using low-level (from viewpoint of their theoretical justification) heuristic models and schemes [3] – [4].

Therefore an important scientific and engineering problem is developing sufficient mathematical models that can formalize the process of QoS ensuring within solving the traffic control task with taking into account multipath routing and AQM mechanisms on routers. In this regard, approach based on the tensor representation of the telecommunication network deserves attention. This mathematical tool has already proven itself to provide effective holistic and multi-aspect description of telecommunication network. In [5] – [6] tensor model of TCN enables to obtain analytical conditions for satisfaction rate and delay requirements at same time under multipath routing.

Providing a required level of reliability of traffic delivery is related to using measurements such as the probability of timely delivery of packet, the probability of authentic delivery of packet, the probability of failure-free operation, availability factor, etc. Reference [7] develops reliability tensor model of TCN in terms of the probability of failure-free operation. In this article we'll focus on the probability of packet loss (IP packet Loss Ratio, IPLR), which is one of the key characteristics of Network Performance [8].

II. TENSOR MODEL OF THE TELECOMMUNICATION NETWORK WITH RED

In order to develop tensor model of TCN we'll use a technique based on the generalization postulates of G. Kron [9]. According to a preliminary postulate in the first phase of development behavior equation for a single element of the system should be written. Let us choose link as single element of telecommunication network. Then we'll consider the network as a set of connected in a certain way (within a certain structure) links.

It is known that the delivery of the packet in the link and the loss of the packet form a complete group of events, i.e.

$$p = 1 - p_l, \quad (1)$$

where p – the probability of packet delivery; p_l – the probability of packet loss.

In general, the causes of a packet loss can be different, for example, the signal's distortion, coding errors, incorrect addressing, a large network delay and expiration of TTL of the packet. But the main reason of packet loss in transport network is related to a buffer overflow and packet drops, i.e. mechanisms of passive and active queue management. At

Manuscript received November 23, 2012.

Olexandr Lemeshko is with the Kharkov National University of Radioelectronics, Ukraine, 61166, Kharkov, Lenin Prosp., 14, room 305 (corresponding author to provide phone: (057)7021320; fax: (057)7021320; e-mail: avlem@mail.ru).

Oksana Yevsyeyeva is with the Kharkov National University of Radioelectronics, Ukraine, 61166, Kharkov, Lenin Prosp., 14, room 305 (phone: (057)7021320; fax: (057)7021320; e-mail: evseeva.o.yu@gmail.com).
Sergey Garkusha is with the Kharkov National University of Radioelectronics, Ukraine, 61166, Kharkov, Lenin Prosp., 14, room 305 (phone: (057)7021320; fax: (057)7021320; e-mail: sv.garkuha@mail.ru).

present, most widely applicable queue management mechanisms in the packet-switched networks are Random Early Detection (RED) and its different modifications [2], [4]. RED and its modifications belong to AQM mechanisms where dropping of packets from queue can happen before buffer overflow. Under RED, discard function p_l is linear function of the average length of the queue q

$$p_l = \frac{q - \Theta_{\min}}{\Theta_{\max} - \Theta_{\min}} \cdot \frac{1}{\delta}, \quad (2)$$

where q – actual size of the queue (number of packets in queue); Θ_{\min} – minimum threshold (if the average queue falls below this minimum threshold then no packets are discarded); Θ_{\max} – maximum threshold (if the falls above this maximum threshold all packets are discarded); δ – mark probability denominator.

Then delivery probability for link under RED is

$$p = 1 - \frac{q - \Theta_{\min}}{\Theta_{\max} - \Theta_{\min}} \cdot \frac{1}{\delta}. \quad (3)$$

The average length of queue q is function of traffic intensity transmitted through the router (link) for the formalization of which we will use the results of queuing theory as one possible way of its analytical representation. By using queuing system $M/M/1/N$ this quantity can be represented as [10]

$$q = \frac{\rho}{1 - \rho} - \frac{(N+1)\rho^{N+1}}{1 - \rho^{N+1}} - \rho. \quad (4)$$

where $N = \Theta_b + 1$; Θ_b – buffer size; $\rho = \frac{\lambda}{\varphi}$ – utilization of the link; φ – capacity of the link; λ – traffic rate in the link.

Note, as well as in the case of the functional equation for a single network element, the estimated average queue length can be obtained not only by using the queuing theory, but also by using other mathematical tools such as Markov processes, empirical methods, etc.

Let us add a sliding index i for indicating the number of the link, then the functional equation for reliability of the i th link can be written as

$$p_i^{(v)} = 1 - \frac{\left(\frac{\rho_i}{1 - \rho_i} - \frac{(N_i + 1)\rho_i^{N_i + 1}}{1 - \rho_i^{N_i + 1}} - \rho_i \right) - \Theta_{\min}}{\Theta_{\max} - \Theta_{\min}} \cdot \frac{1}{\delta}, \quad (5)$$

where $p_i^{(v)}$ – the probability of packet delivery through i th link, $i = \overline{1, n}$; (v) – mute index (indicates that the link belongs to set of edges V in graph model of network) [11]; ρ_i – utilization of the i th link; n – number of link in the network.

The system of equations (5) describes separated network links. Before turning the system of equations into one tensor equation we must be sure that every object from the system (5) has tensor nature. References [5] – [6] show tensor nature of some network parameters (metrics). It is known that additive metrics such as delay, jitter are covariant tensors but metrics satisfying conservation constraint, for instance, traffic intensity or rate, are contravariant tensors.

The probability of packet delivery is multiplicative metric, i.e.

$$p^{(path)} = \prod_{i: v_i \in path} p_i^{(v)}.$$

Let us turn the multiplicative metric into the additive

$$\log_a(p^{(path)}) = \sum_{i: v_i \in path} \log_a(p_i^{(v)}).$$

Then (5) takes the form

$$\log_a(p_i^{(v)}) = \left[\frac{1}{\lambda_{(v)}^i} \times \right. \\ \left. \times \log_a \left(1 - \frac{\left(\frac{\rho_i}{1 - \rho_i} - \frac{(N_i + 1)\rho_i^{N_i + 1}}{1 - \rho_i^{N_i + 1}} - \rho_i \right) - \Theta_{\min}}{\Theta_{\max} - \Theta_{\min}} \cdot \frac{1}{\delta} \right) \right] \lambda_{(v)}^i \quad (6)$$

or

$$P_v = \Theta_v \Lambda_v, \quad (7)$$

where N_i and $\lambda_{(v)}^i$ belong to i th link; P_v – vector of reduced (turned into additive form) probability of packet delivery with elements $\log_a(p_i^{(v)})$; Θ_v – diagonal matrix with elements

$$\theta_{ii}^{(v)} = \frac{1}{\lambda_{(v)}^i} \log_a \left(1 - \frac{\left(\frac{\rho_i}{1-\rho_i} - \frac{(N_i+1)\rho_i^{N_i+1}}{1-\rho_i^{N_i+1}} - \rho_i \right) - \Theta_{\min}}{(\Theta_{\max} - \Theta_{\min})\delta} \right). \quad (8)$$

Equation (7) can be interpreted as a projection of the following invariant (tensor) equation in the coordinate system (CS) of edges (type v)

$$\mathbf{P} = \Theta \mathbf{A}, \quad (9)$$

where \mathbf{P} – univalent covariant tensor of the reduced probability of packet delivery; \mathbf{A} – univalent contravariant tensor of traffic intensity; Θ – divalent covariant tensor acting as a metric tensor.

Equation (9) can be written as

$$\mathbf{A} = \mathbf{X} \mathbf{P}, \quad (10)$$

where \mathbf{X} – divalent contravariant metric tensor, whose projection in the CS of edges is $X_v = [\Theta_v]^{-1}$.

Note that coordinate system of edges considers the network as a set of separated links, i.e. set of single edges.

Thus, probability tensor model of TCN can be reduced to an invariant tensor equation (9), where coordinates of the divalent covariant tensor Θ (8) in the CS of edges are functions of the discarding parameters (Θ_{\min} , Θ_{\max} , δ), the size of buffer (Θ_b), the capacities of the links (φ_i), and the intensities of the traffic transmitted through the routers ($\lambda_{(v)}^i$).

III. FORMULATION OF CONDITION FOR ENSURING REQUIRED RELIABILITY OF SERVICE

In order to derive the condition for ensuring quality in terms of reliability we'll use orthogonal representation of the tensor model of TCN (9) – (10) in CS of circuits and pairs of nodes. This CS considers the network as a set of circuits π and node pairs η , where total dimension of CS is equal to n . Then the projections of tensors of traffic intensity \mathbf{A} and the reduced probability of delivery \mathbf{P} in this coordinate system can be represented by the following vectors:

$$\Lambda_{\pi\eta} = \begin{Bmatrix} \Lambda_{\pi} \\ \Lambda_{\eta} \end{Bmatrix}, \quad \Lambda_{\pi} = \begin{Bmatrix} \lambda_{(\pi)}^1 \\ \vdots \\ \lambda_{(\pi)}^j \\ \vdots \\ \lambda_{(\pi)}^{\mu} \end{Bmatrix}, \quad \Lambda_{\eta} = \begin{Bmatrix} \lambda_{(\eta)}^1 \\ \vdots \\ \lambda_{(\eta)}^j \\ \vdots \\ \lambda_{(\eta)}^{\phi} \end{Bmatrix}, \quad (11)$$

$$P_{\pi\eta} = \begin{Bmatrix} P_{\pi} \\ P_{\eta} \end{Bmatrix}, \quad P_{\pi} = \begin{Bmatrix} p_1^{(\pi)} \\ \vdots \\ p_j^{(\pi)} \\ \vdots \\ p_{\mu}^{(\pi)} \end{Bmatrix}, \quad P_{\eta} = \begin{Bmatrix} p_1^{(\eta)} \\ \vdots \\ p_j^{(\eta)} \\ \vdots \\ p_{\phi}^{(\eta)} \end{Bmatrix}, \quad (12)$$

where $\Lambda_{\pi\eta}$, $P_{\pi\eta}$ – n -dimensional vectors that are projections of tensors \mathbf{P} and \mathbf{A} in CS of circuits and node pairs; Λ_{π} , P_{π} – μ -dimensional subvectors related to circuits in network, $\mu = n - m + 1$; m – number of nodes in the network; Λ_{η} , P_{η} – ϕ -dimensional subvectors related to node pairs in network, $\phi = m - 1$.

Note that circuit components $\lambda_{(\pi)}^j$ и $p_j^{(\pi)}$ from subvectors Λ_{π} and P_{π} are related to circuits in a network. So in order to eliminate loops in routes we must satisfy the next condition

$$P_{\pi} = 0. \quad (13)$$

The components of subvectors Λ_{η} and P_{η} show traffic intensity and the reduced probability of delivery for different pairs of nodes in a network. Then according flow conservation law for every transit nodes value $\lambda_{(\eta)}^j$ must be zero:

$$\Lambda_{\eta} = \begin{Bmatrix} \lambda_{(\eta)}^1 & 0 & \dots & 0 \end{Bmatrix}^t, \quad (14)$$

where $\lambda_{(\eta)}^1$ – traffic intensity between end points which form first pair of nodes.

In accordance with the second generalization postulate of G. Kron [9] tensor equations (9) and (10) have the same form in every the coordinate system, i.e. in CS of circuits and node pairs tensor equation (10) takes the form

$$\Lambda_{\pi\eta} = X_{\pi\eta} P_{\pi\eta}, \quad (15)$$

where $X_{\pi\eta}$ – projection of tensor \mathbf{X} in CS of circuits and node pairs.

According to laws of tensor calculus projections of tensors \mathbf{P} , \mathbf{A} and \mathbf{X} in the CS of circuits and node pairs (type $\pi\eta$) and in the CS of edges (type v) are related as follows

$$P_v = A P_{\pi\eta}, \quad (16)$$

$$\Lambda_v = C \Lambda_{\pi\eta}, \quad (17)$$

$$X_v = C X_{\pi\eta} C^t, \quad (18)$$

$$X_{\pi\eta} = A^t X_v A. \quad (19)$$

where A and C – matrices of co- and contravariant transformation of coordinates when transition from CS of circuits and node pairs to CS of edges.

Using (11) – (123) we can represent (15) in next form

$$\begin{pmatrix} \Lambda_\pi \\ \Lambda_\eta \end{pmatrix} = \begin{pmatrix} X_{\pi\eta}^{(1)} & | & X_{\pi\eta}^{(2)} \\ X_{\pi\eta}^{(3)} & | & X_{\pi\eta}^{(4)} \end{pmatrix} \cdot \begin{pmatrix} P_\pi \\ P_\eta \end{pmatrix}, \quad (20)$$

where $\begin{pmatrix} X_{\pi\eta}^{(1)} & | & X_{\pi\eta}^{(2)} \\ X_{\pi\eta}^{(3)} & | & X_{\pi\eta}^{(4)} \end{pmatrix} = X_{\pi\eta}$, $X_{\pi\eta}^{(1)}$, $X_{\pi\eta}^{(4)}$ – square $\mu \times \mu$

and $\phi \times \phi$ submatrices, respectively; $X_{\pi\eta}^{(2)}$ – $\mu \times \phi$

submatrix, $X_{\pi\eta}^{(3)}$ – $\phi \times \mu$ submatrix.

Then from (20) and according (13) we have

$$\Lambda_\eta = X_{\pi\eta}^{(4)} P_\eta. \quad (21)$$

Further we will consider vectors Λ_η and P_η as

$$\Lambda_\eta = \begin{pmatrix} \lambda_{(\eta)}^1 \\ \Lambda_{\eta-1} \end{pmatrix}, \quad P_\eta = \begin{pmatrix} p_1^{(\eta)} \\ P_{\eta-1} \end{pmatrix}, \quad \text{where } p_1^{(\eta)} \text{ – reduced}$$

probability of traffic delivery between end points which form first pair of nodes. Then (21) can be turned into

$$\begin{pmatrix} \lambda_{(\eta)}^1 \\ \Lambda_{\eta-1} \end{pmatrix} = \begin{pmatrix} X_{\pi\eta}^{(4,1)} & | & X_{\pi\eta}^{(4,2)} \\ X_{\pi\eta}^{(4,3)} & | & X_{\pi\eta}^{(4,4)} \end{pmatrix} \cdot \begin{pmatrix} p_1^{(\eta)} \\ P_{\eta-1} \end{pmatrix}, \quad (22)$$

where $\begin{pmatrix} X_{\pi\eta}^{(4,1)} & | & X_{\pi\eta}^{(4,2)} \\ X_{\pi\eta}^{(4,3)} & | & X_{\pi\eta}^{(4,4)} \end{pmatrix} = X_{\pi\eta}^{(4)}$, $X_{\pi\eta}^{(4,1)}$ – the first

element of the matrix $X_{\pi\eta}^{(4)}$.

From (14) and (22) we obtain

$$\lambda_{(\eta)}^1 = \left(X_{\pi\eta}^{(4,1)} - X_{\pi\eta}^{(4,2)} \left[X_{\pi\eta}^{(4,4)} \right]^{-1} X_{\pi\eta}^{(4,3)} \right) p_1^{(\eta)}. \quad (23)$$

Elements $\lambda_{(\eta)}^1$ and $p_1^{(\eta)}$ are related to pair source-destination and in general can include requirements for traffic intensity (rate) and the reduced probability of delivery

(reliability) for this pair, i.e. $\lambda_{(\eta)}^1 = \lambda^{(req)}$, $p_1^{(\eta)} = p_{(req)}$,

$P_{(req)} = \log_a \left(1 - P_{IPLR}^{(req)} \right)$, $P_{IPLR}^{(req)}$ – required value of IPLR.

Then finally we have the following inequality

$$\lambda^{(req)} \geq \left(X_{\pi\eta}^{(4,1)} - X_{\pi\eta}^{(4,2)} \left[X_{\pi\eta}^{(4,4)} \right]^{-1} X_{\pi\eta}^{(4,3)} \right) p_{(req)}, \quad (24)$$

which is a formalization of the condition for ensuring the required quality of service between a given pair of recipients from reliability point of view. It is assumed that this condition can be placed into dynamic or static model of TCN for solving traffic control (engineering) problem in networks with guaranteed QoS.

IV. EXAMPLE OF THE SOLUTION OF THE QOS-ENSURING PROBLEM WITH RATE AND RELIABILITY REQUIREMENTS

Let us make an example of solving QoS-ensuring problem with two required parameters (transmission rate and the probability of packet delivery) in environment of multipath routing and active queue management mechanism such as RED. The solving QoS-ensuring problem is related to traffic distribution under which given QoS-requirements will be satisfied. Fig. 1 shows example of network where capacity φ_i for every link is known (Table I). For given pair source-destination we will find set of routes such as total for traffic rate (intensity) from source to destination will be not less than $\lambda^{(req)} = 535$ 1/s (in packets per second) and result loss will be not more than $P_{IPLR}^{(req)} = 0,03$ ($p_{(req)} = \log_2(0,97) = -0,0439$).

To simplify the problem, assume parameters of mechanism RED on all nodes are the same: $\Theta_{\min} = 5$ and $\Theta_{\max} = 40$ packets, $\delta = 10$, which correspond to the recommended parameters.

Numerical results that satisfy the condition (24) and given QoS-requirements are shown into Table I and in Fig. 2. According to the results for servicing traffic between nodes 1 (source) and 6 (destination) at given rate $\lambda^{(req)}$ and with given IPLR $P_{IPLR}^{(req)}$ we need use four paths that are shown in Fig. 1 and into Table II.

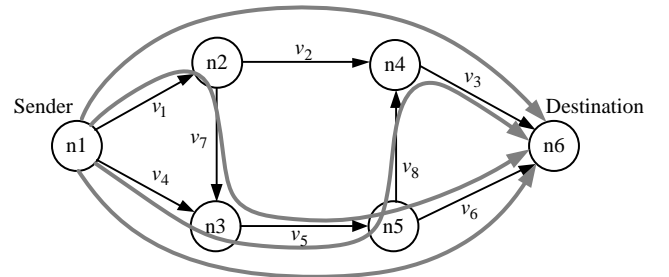


Fig. 1. Example of network structure and obtained set of paths

TABLE I
RESULT TRAFFIC DISTRIBUTION AS SOLUTION OF QoS-ENSURING PROBLEM

Number of the link (edge) i	Capacity of the link $\varphi_i, 1/s$	Traffic intensity in the link $\lambda_{(v)}^i, 1/c$	Reduced probability of delivery $\log_2(p_i^{(v)})$	Probability of delivery in the link $p_i^{(v)}$	Probability of loss in the link $1 - p_i^{(v)}$
1	445	401	-0,0116	0,9920	0,0080
2	282	265	-0,0268	0,9816	0,0184
3	432	381	-0,0055	0,9962	0,0038
4	147	134	-0,0155	0,9893	0,0107
5	292	270	-0,0195	0,9866	0,0134
6	172	154	-0,0089	0,9939	0,0061
7	155	136	-0,0039	0,9973	0,0027
8	133	116	-0,0034	0,9977	0,0023

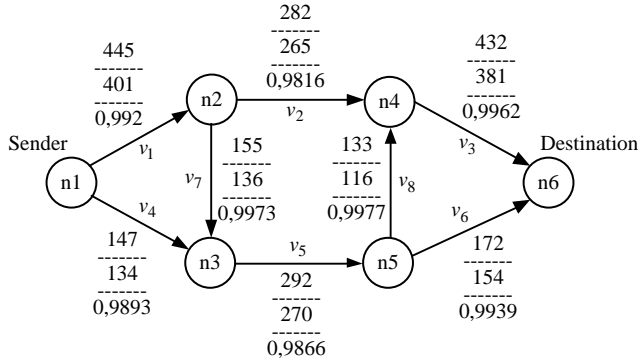


Fig. 2. Result traffic distribution between links that satisfy given QoS-requirements. Values near every link show (top-down) capacity, traffic intensity and the probability of delivering through the link.

TABLE II
SET OF USED ROUTES AND PROBABILITY OF DELIVERY THROUGH ITS

Path	Traffic intensity through path, 1/s	Probability of delivery through path
$\langle v_1, v_2, v_3 \rangle$	≈ 265	$(0,992 \cdot 0,9816 \cdot 0,9962) \approx 0,97$
$\langle v_1, v_5, v_6, v_7 \rangle$	≈ 136	$(0,992 \cdot 0,9866 \cdot 0,9939 \cdot 0,9973) \approx 0,97$
$\langle v_4, v_5, v_6 \rangle$	≈ 18	$(0,9893 \cdot 0,9866 \cdot 0,9939) \approx 0,97$
$\langle v_3, v_4, v_5, v_8 \rangle$	≈ 116	$(0,9962 \cdot 0,9893 \cdot 0,9866 \cdot 0,9977) \approx 0,97$

Thus, the obtained solution, firstly, meets the specified rate and reliability (packet loss) requirements, and secondly, provides not only desired, but the same probability of delivery (loss) through all traffic routes.

V. CONCLUSIONS

Thus, the problem of traffic control with QoS-ensuring requires an adequate mathematical model of the telecommunication network that take into account the QoS requirements, on the one hand, and the structural and functional features of the TCN and characteristics of traffic, on the another hand. Such contradictory requirements can be satisfied within tensor approach, which has been demonstrated in this paper. The obtained formalization of the condition for ensuring the required reliability of service in networks takes into account characteristics of traffic, the parameters of the active queue management mechanisms, the structural properties of the network and focuses on the multipath transmitting. The condition (24) was formulated from

invariant tensor equations and has invariant form that doesn't depend on AQM mechanism type. The parameters of AQM affect numerical values of the projections of the metric tensor, and doesn't affect the form of the condition (24). This distinctive feature allows to apply the condition (24) in the network not only with RED, but with other mechanisms of active queue management.

REFERENCES

- [1] General overview of NGN, ITU-T Recommendation Y.2001, 12/2004.
- [2] Kun I. Park, *QoS in Packet Networks*. Springer, 2004.
- [3] Yanxing Zheng, Wenhua Dou, Jing Tian, Mingyan Xiao "An Overview of Research on QoS Routing" in *Advanced Parallel Processing Technologies*, Lecture Notes in Computer Science, vol. 2834, Zhou, X.; Jähnichen, S.; Xu, M.; Cao, J., Eds., Springer, 2003, pp 387-397
- [4] R. Adams, "Active Queue Management: A Survey", *IEEE Communications Surveys & Tutorials*, vol. PP, issue 99, pp. 1 – 52, 2012.
- [5] O. Lemeshko, O. Yevsyeyeva "Tensor geometrization of structural and functional representation of telecommunication system in the basis of interpolates paths and internal cuts", *Scientific notes of UNDIZ*, n. 1 (13). pp. 14 – 26, 2010. (in Russian)
- [6] O. Lemeshko "Tensor model of multipath routing of aggregated flows with reservation network resources, that is provided in a curvature space", *Transactions of UNDIRT*, n. 4 (40), pp. 12 – 18, 2004. (in Russian)
- [7] A. V. Lemeshko, "Probabilistic-Temporal Model of QoS-Routing with Precomputation of Routes under the Terms of Non-Ideal Reliability of Telecommunication Network," *Telecommunications and Radio Engineering*, vol. 66, issue 13, pp. 1151-1166.
- [8] Network performance objectives for IP-based services, ITU-T Recommendation Y.1541, 05/2002.
- [9] G. Kron, *Tensors for Circuits*. Dover Publ., New York, 1959.
- [10] D. Gross, J. F. Shortle, J. M. Thompson, C. M. Harris, *Fundamentals of Queueing Theory*. John Wiley & Sons, 2008.
- [11] O. Yevsyeyeva "Classification of spaces and bases used in the geometric description of the structure of telecommunication network", *Radiotekhnika*, n. 159, pp. 14 – 19, 2009. (in Russian)